Instability due to the dust-particulate–phonon interaction

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Interactions of dust particulates and phonons in a plasma are studied from a quantum mechanical viewpoint. An attraction between negatively charged dust particulates due to a stable wake potential behind a dust particulate in the supersonic ion flow is shown to be the result of exchanging phonons between the dust particulates. A collection of dust particulates becomes hydrodynamically unstable when the ion flow becomes subsonic. The growth rate of the instability is given by $\gamma=0.87\ (\omega_{\rm pi}/2\omega_{\rm pd})^{1/3}\Omega_{\mathbf{k}}$, where $\omega_{\rm pi}$ is the ion plasma frequency, $\omega_{\rm pd}$ is the dust plasma frequency, and $\Omega_{\mathbf{k}}$ is the dust acoustic frequency. [S1063-651X(98)02609-9]

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I. INTRODUCTION

It was in 1952 when Pines and Bohm showed the formation of wake potential trailing behind a charged particle when it moves through an electron gas [1]. The effect is a manifestation of collective effects in plasmas. The energy given up by the incident particle is transferred to the wake potential, which is characterized by the oscillatory nature in space. More than 40 years later, the theory of wake potential is applied to understand the formation of plasma crystals observed in laboratory experiments [2–7]. The wake potential effect was derived mathematically by an analysis of the contour integral involved in a potential calculation. The electrostatic potential around a dust particulate of charge Q, placed at $\mathbf{x}=\mathbf{x}_0$ at $t=0$ and moving with a velocity $\mathbf{v}$ in a plasma of volume V, is given by [6]

$$\phi(\mathbf{x},t)=\frac{4\pi Q}{V}\sum_{\mathbf{k}}\int_{Br}\frac{d\omega}{2\pi}\frac{\exp[i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}_0)-\omega t]}{-i(\omega-\mathbf{k}\cdot\mathbf{v}+i\eta)k^2\epsilon(\mathbf{k},\omega)},\tag{1}$$

where ω integration is defined for the Bromwich integral, η is the positive infinitesimal, $\epsilon(\mathbf{k},\omega)$ is the dielectric response function of the plasma, and $\Sigma_{\mathbf{k}}=V\int d^3k/(2\pi)^3$ in the limit of $V\to\infty$. If we replace the response function by the static form factor $\epsilon(\mathbf{k},0)=1+1/k^2\lambda_D^2$ (λ_D is the Debye wavelength), and set $\mathbf{x}_0=\mathbf{v}=0$, we recover the conventional Debye shielded potential

$$\phi(\mathbf{x})=\frac{Q}{r}e^{-r/\lambda_D},\tag{2}$$

where $r=|\mathbf{x}|$. When the plasma has an ion flow and supports ion acoustic waves, the response function is

$$\epsilon(\mathbf{k},\omega)=1+\frac{1}{k^2\lambda_D^2}-\frac{\omega_{\rm pi}^2}{(\omega-\mathbf{k}\cdot\mathbf{v}_0)^2}.\tag{3}$$

Here $k=|\mathbf{k}|$, and $\omega_{\rm pi}$ is the ion plasma frequency. The contour integration of Eq. (1) gives the oscillatory potential behind the dust particulate placed at the origin in cylindrical coordinates (ρ,z,θ) with velocity $\mathbf{v}=v\mathbf{e}_z(v\geqslant 0)$ in the presence of ion flow $\mathbf{v}_0=v_0\mathbf{e}_z(v_0<0)$ [6]:

$$\phi(\rho,z,t)=\frac{2Q\lambda_D^2M^2}{(M^2-1)^{3/2}}\int_0^{1/\lambda_D}dk\ k^2J_0(k\rho)\sin\left(\frac{k(z-vt)}{\sqrt{M^2-1}}\right)\tag{4}$$

for

$$v_0t<z<vt\qquad\text{and}\qquad|z-vt|>\rho\sqrt{M^2-1},\tag{5}$$

where J_0 is the zeroth-order Bessel function of the first kind, $M=|v-v_0|/C_s$, $C_s=\sqrt{T_e/m_i}$ is the ion acoustic velocity (T_e is the electron temperature, and m_i the ion mass), and cylindrical symmetry is assumed. We note here that the wake potential is formed only for $M>1$, or supersonic ion flow, which is realized in a sheath region [8]. Plasma crystals, first observed in low power radio-frequency discharges in 1994 [9], are Coulomb lattices formed by negatively charged micron-size dust particulates floating in the balance of gravitation and electrostatic forces in a plasma. The wake potential is formed by a highly negative dust particulate (Q ranges from -10^3e to -10^4e, where e is the magnitude of electron charge) floating in the presence of ambient ion flow, and contributes to attract a like charge dust particulate.

Plasma crystals are characterized by their ordered structure with polygon forms of a crystalline system. Such a well established crystalline structure was observed to be stable, but to experience a transition to a disordered liquidlike structure by lowering the neutral gas pressure in low-power radio-frequency discharges [10,11]. Low frequency oscillations on the order of 10 Hz were observed before the melting transition occurs, and became a topic of investigation [12–15]. The stability of an ordered polygon structure was also discussed, including the damping effect due to dust particulate-neutral collisions [16,17] and the viscous effect due to ion drags [18].

In this paper, the interaction of dust particulates in a plasma which supports ion acoustic waves is studied from a quantum mechanical viewpoint. A quantum mechanical treatment applied to interaction between plasma particles and waves has helped to understand the physics of some classical phenomena [19–21]. For example, the Landau damping, which results from the interaction of plasma waves with

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resonant plasma particles, is interpreted as a result of the competing effect between emission and absorption of plasmons (quasiparticles for plasma waves) by plasma particles. Equation (1), together with Eq. (4), suggests that the source of the potential deviation from the conventional Debye shielded Coulomb potential is collective effects of the plasma, or the presence of phonons. A Hamiltonian study of dust-particulate–plasma interaction by the method of canonical transformation revealed that the wake potential is a result of the interaction between two dust particulates exchanging phonons (the quanta of ion acoustic waves) [7]. The two dust particulates, both with negative charges, will behave like Cooper pairs in the theory of superconductivity. In this paper an interaction Hamiltonian for dust particulates and phonons is derived based on the second quantization formalism. The Hamiltonian thus derived agrees with the one derived earlier by the method of canonical transformation, but the present, rather simple, derivation based on a vertex matrix calculation gives a straightforward understanding of the resulting attraction between like charged dust particulates. The matrix element is then used to discuss the collective behavior of dust particulates which interact among themselves by exchanging phonons. Simple hydrodynamic equations describe the motion of the gas in the effective potential described by the matrix element. We find that the system becomes unstable and grows with time in inverse proportion to the dust acoustic frequency, with a certain factor depending on situations including Coulomb interaction and ion flow.

In Sec. II, we introduce the quantization of the electric fields associated with ion acoustic waves, and derive the interaction Hamiltonian of quasiparticles (phonons) and dust particulates. In Sec. III, the interaction among dust particulates is treated as a scattering of dust particulates in phonon fields, and the emission and absorption of phonons are considered. Section IV deals with the hydrodynamic behavior of dust particulates in the effective potential described by the interaction matrix element. The conclusion is given in Sec. V.

II. DUST-PARTICULATE–PHONON INTERACTION

We consider dust particulates in the potential perturbations associated with ion acoustic waves in a plasma. We describe the perturbations in terms of quasiparticles called phonons. The phonon fields may be described by the potential $\phi(\mathbf{x},t)$ in a Fourier series in a large box of volume V as

$$\phi(\mathbf{x},t)=\sum_{\mathbf{k}} \sqrt{\frac{2\pi\hbar\omega_{\mathbf{k}}\lambda_D^2}{V(1+k^2\lambda_D^2)}}\,[a_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)} + a_{\mathbf{k}}^{\dagger}e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}], \tag{6}$$

where $k=|\mathbf{k}|$, $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$ are Fourier coefficients, which may be interpreted, as is known in second quantization, as destruction and creation operators of phonons with momentum $\hbar\mathbf{k}$ and energy $\hbar\omega_{\mathbf{k}}$. A positive frequency of an ion acoustic wave is given by

$$\omega_{\mathbf{k}}=\frac{kC_s}{\sqrt{1+k^2\lambda_D^2}}. \tag{7}$$

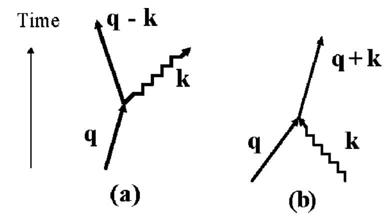

FIG. 1. Dust-particulate–phonon interaction through (a) emission or (b) absorption of a phonon.

The factor in Eq. (6) is set to make contact with the quantum mechanical expression of the electric field energy (see the Appendix), or

$$\int d^3x\,\frac{\langle|\nabla\phi|^2\rangle}{8\pi}=\sum_{\mathbf{k}}\frac{\hbar\omega_{\mathbf{k}}}{2\left|\frac{\partial}{\partial\omega}\,\omega\epsilon\right|_{\omega_{\mathbf{k}}}}(a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}+a_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}). \tag{8}$$

We introduce the wave function for dust particulates as

$$\Psi=\frac{1}{\sqrt{V}}\sum_{\mathbf{q}} C_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{x}}, \tag{9}$$

and interpret Ψ as an operator to destroy a dust particulate of momentum $\hbar\mathbf{q}$ at position $\mathbf{x}$. The operators $C_{\mathbf{q}}$ and $C_{\mathbf{q}}^{\dagger}$ are destruction and creation operators of dust particulates carrying the momentum $\hbar\mathbf{q}$. A dust particulate with charge Q interacts with longitudinal phonon fields, and the interaction Hamiltonian between a dust particulate and a phonon is given by

$$\mathcal{H}_{d,p}=\int \Psi^{\dagger}Q\phi\Psi\; d^3x, \tag{10}$$

which can be simplified as

$$\mathcal{H}_{d,p}=\sum_{\mathbf{k}}\sum_{\mathbf{q}} M_{d,p}(\mathbf{k})(C_{\mathbf{q}-\mathbf{k}}^{\dagger}a_{\mathbf{k}}^{\dagger}C_{\mathbf{q}}e^{i\omega_k t}+C_{\mathbf{q}+\mathbf{k}}^{\dagger}a_{\mathbf{k}}C_{\mathbf{q}}e^{-i\omega_k t}), \tag{11}$$

where

$$M_{d,p}(\mathbf{k})=\sqrt{\frac{4\pi Q^2\hbar\omega_{\mathbf{k}}}{Vk^2\left|\frac{\partial}{\partial\omega}\omega\epsilon\right|_{\omega_{\mathbf{k}}}}}=\sqrt{\frac{2\pi Q^2\hbar\omega_{\mathbf{k}}}{V(k^2+\lambda_D^{-2})}}. \tag{12}$$

Here we have used $|(\partial/\partial\omega)\omega\epsilon|_{\omega_{\mathbf{k}}}=2(1+k^{-2}\lambda_D^{-2})$. The term involving $C_{\mathbf{q}-\mathbf{k}}^{\dagger}a_{\mathbf{k}}^{\dagger}C_{\mathbf{q}}$ describes the scattering of a dust particulate from $\mathbf{q}$ to $\mathbf{q}-\mathbf{k}$ with the emission of a phonon of momentum $\hbar\mathbf{k}$, while the term involving $C_{\mathbf{q}+\mathbf{k}}^{\dagger}a_{\mathbf{k}}C_{\mathbf{q}}$ describes the scattering of a dust particulate from $\mathbf{q}$ to $\mathbf{q}+\mathbf{k}$ with the absorption of a phonon of momentum $\hbar\mathbf{k}$. These interactions are shown in Fig. 1.

III. INTERACTION OF DUST PARTICULATES BY PHONON EXCHANGE

A higher order approximation in the perturbation theory gives the transition matrix, which describes the transition from the initial state $|i\rangle$ to the final state $|f\rangle$, as [23]

$$M_{if}=\langle f|H_I|i\rangle+\sum_I \frac{\langle f|H_I|I\rangle\langle I|H_I|i\rangle}{E_i-E_I+i\eta}, \tag{13}$$

where $H_I=Q\phi$, E_i and E_I are the initial and the intermediate energies, the notation $\langle f|H_I|i\rangle$ is used for $\int\Psi_f^\dagger H_I\Psi_i d^3x$, $|I\rangle$ is the intermediate state through which the transition takes place, the summation is taken for all the possible intermediate states, and the quantity η is a positive infinitesimal. The second term in Eq. (13) is known to be responsible for the attractive interaction between electrons associated with the superconducting state [24,25]. Here we calculate the second term for the interaction of dust particulates through the exchange of phonons. Suppose that the initial, intermediate, and final wave functions for dust particulates are given by

$$\Psi_i=\frac{1}{\sqrt{V}}C_{\mathbf{q}_i}e^{i\mathbf{q}_i\cdot\mathbf{x}},\quad \Psi_I=\frac{1}{\sqrt{V}}C_{\mathbf{q}_I}e^{i\mathbf{q}_I\cdot\mathbf{x}},$$

$$\Psi_f=\frac{1}{\sqrt{V}}C_{\mathbf{q}_f}e^{i\mathbf{q}_f\cdot\mathbf{x}}. \tag{14}$$

Then the matrix elements are

$$\langle I|H_I|i\rangle=\frac{1}{V}\int d^3x\sum_{\mathbf{k}} M_{d,p}(\mathbf{k})C_{\mathbf{q}_I}^\dagger e^{-i\mathbf{q}_I\cdot\mathbf{x}}\{a_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}+a_{\mathbf{k}}^\dagger e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}\}C_{\mathbf{q}_i}e^{i\mathbf{q}_i\cdot\mathbf{x}} \tag{15}$$

or

$$\langle I|H_I|i\rangle=\sum_{\mathbf{k}} M_{d,p}(\mathbf{k})[C_{\mathbf{q}_i+\mathbf{k}}^\dagger a_{\mathbf{k}}C_{\mathbf{q}_i}e^{-i\omega_{\mathbf{k}}t}+C_{\mathbf{q}_i-\mathbf{k}}^\dagger a_{\mathbf{k}}^\dagger C_{\mathbf{q}_i}e^{i\omega_{\mathbf{k}}t}]. \tag{16}$$

Likewise,

$$\langle f|H_I|I\rangle=\sum_{\mathbf{k}'} M_{d,p}(\mathbf{k}')[C_{\mathbf{q}_f}^\dagger a_{\mathbf{k}'}C_{\mathbf{q}_f-\mathbf{k}'}e^{-i\omega_{\mathbf{k}'}t}+C_{\mathbf{q}_f}^\dagger a_{\mathbf{k}'}^\dagger C_{\mathbf{q}_f+\mathbf{k}'}e^{i\omega_{\mathbf{k}'}t}]. \tag{17}$$

Taking the time average, using the relation $\langle e^{\pm i(\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2})t}\rangle=0$, and keeping only the term $\mathbf{k}'=\mathbf{k}$ in the $\mathbf{k}'$ summation to insure a common intermediate state, we obtain

$$\langle f|H_I|I\rangle\langle I|H_I|i\rangle=\sum_{\mathbf{k}}[M_{d,p}(\mathbf{k})]^2[C_{\mathbf{q}_f}^\dagger a_{\mathbf{k}}C_{\mathbf{q}_f-\mathbf{k}}C_{\mathbf{q}_i-\mathbf{k}}^\dagger a_{\mathbf{k}}^\dagger C_{\mathbf{q}_i}+C_{\mathbf{q}_f}^\dagger a_{\mathbf{k}}^\dagger C_{\mathbf{q}_f+\mathbf{k}}C_{\mathbf{q}_i+\mathbf{k}}^\dagger a_{\mathbf{k}}C_{\mathbf{q}_i}], \tag{18}$$

where the first term shows that a dust particulate changes its momentum from $\hbar\mathbf{q}_i$ to $\hbar(\mathbf{q}_i-\mathbf{k})$ by emitting a phonon of momentum $\hbar\mathbf{k}$ which is absorbed by another dust particulate with momentum change from $\hbar(\mathbf{q}_f-\mathbf{k})$ to $\hbar\mathbf{q}_f$, while the second term shows that a dust particulate changes its momentum from $\hbar\mathbf{q}_i$ to $\hbar(\mathbf{q}_i+\mathbf{k})$ and another particulate changes its momentum from $\hbar(\mathbf{q}_f+\mathbf{k})$ to $\hbar\mathbf{q}_f$ by exchanging a phonon of momentum $\hbar\mathbf{k}$. It is assumed that the total number of phonons is conserved in a system. The first term indicates that the absorption of a phonon takes place after the emission, and the reverse occurs in the second term. With the possible two values of $\mathbf{k}$ ($\mathbf{k}$ and $-\mathbf{k}$) in each process, we have four situations as shown in Fig. 2, where a dust particulate with momentum $\hbar\mathbf{q}$ interacts with a dust particulate with momentum $\hbar\mathbf{q}'$. We note that the conservation of energy gives the relation

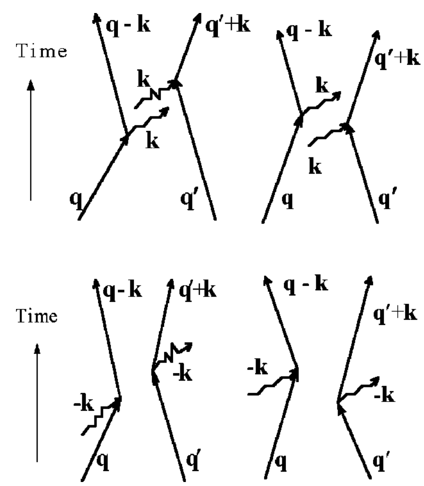

FIG. 2. Interaction of dust particulates through phonon exchange. The number of phonons remains the same before and after the scattering.

$$\mathcal{E}_{\mathbf{q}}+\mathcal{E}_{\mathbf{q}'}=\mathcal{E}_{\mathbf{q}-\mathbf{k}}+\mathcal{E}_{\mathbf{q}'+\mathbf{k}}, \tag{19}$$

where the kinetic energy of a dust particulate of mass m_d is given by

$$\mathcal{E}_{\mathbf{q}}=\frac{|\hbar\mathbf{q}|^2}{2m_d}. \tag{20}$$

The matrix element for the process is thus written as

$$M_{d,d}(\mathbf{k},\mathbf{q},\mathbf{q}')=\sum_I\frac{\langle f|H_I|I\rangle\langle I|H_I|i\rangle}{E_i-E_I+i\eta}=[M_{d,p}(\mathbf{k})]^2\left[\frac{N(\mathbf{k})+1}{\mathcal{E}_{\mathbf{q}}-\mathcal{E}_{\mathbf{q}-\mathbf{k}}-\hbar\omega_{\mathbf{k}}}+\frac{N(\mathbf{k})}{\mathcal{E}_{\mathbf{q}'}-\mathcal{E}_{\mathbf{q}'+\mathbf{k}}+\hbar\omega_{\mathbf{k}}}+\frac{N(-\mathbf{k})}{\mathcal{E}_{\mathbf{q}}-\mathcal{E}_{\mathbf{q}-\mathbf{k}}+\hbar\omega_{-\mathbf{k}}}+\frac{N(-\mathbf{k})+1}{\mathcal{E}_{\mathbf{q}'}-\mathcal{E}_{\mathbf{q}'+\mathbf{k}}-\hbar\omega_{-\mathbf{k}}}\right]. \tag{21}$$

With the help of Eq. (19), the matrix element becomes

$$M_{d,d}(\mathbf{k},\mathbf{q})=[M_{d,p}(\mathbf{k})]^2 \times\left(\frac{1}{\mathcal{E}_{\mathbf{q}}-\mathcal{E}_{\mathbf{q}-\mathbf{k}}-\hbar\omega_{\mathbf{k}}}-\frac{1}{\mathcal{E}_{\mathbf{q}}-\mathcal{E}_{\mathbf{q}-\mathbf{k}}+\hbar\omega_{-\mathbf{k}}}\right), \tag{22}$$

which is further simplified, by using $\omega_{-\mathbf{k}}=\omega_{\mathbf{k}}$ and Eq. (12), as

$$\begin{aligned} M_{d,d}(\mathbf{k},\mathbf{q}) &= \frac{2\hbar\omega_{\mathbf{k}}}{(\mathcal{E}_{\mathbf{q}}-\mathcal{E}_{\mathbf{q}-\mathbf{k}})^2-(\hbar\omega_{\mathbf{k}})^2}[M_{d,p}(\mathbf{k})]^2 \\ &= \frac{4\pi Q^2}{V}\frac{\lambda_D^2}{1+k^2\lambda_D^2}\frac{\omega_{\mathbf{k}}^2}{\omega_{\mathbf{q},\mathbf{k}}^2-\omega_{\mathbf{k}}^2} \\ &= M^W(\mathbf{k},\omega_{\mathbf{q},\mathbf{k}}), \end{aligned} \tag{23}$$

where

$$M^W(\mathbf{k},\omega)=\frac{4\pi Q^2}{V}\frac{\lambda_D^2}{1+k^2\lambda_D^2}\frac{\omega_{\mathbf{k}}^2}{\omega^2-\omega_{\mathbf{k}}^2} \tag{24}$$

and

$$\omega_{\mathbf{q},\mathbf{k}}=\frac{\mathcal{E}_{\mathbf{q}}-\mathcal{E}_{\mathbf{q}-\mathbf{k}}}{\hbar}. \tag{25}$$

The effective Hamiltonian for interaction of dust particulates through the exchange of phonons is

$$H_{d,d}=\sum_{\mathbf{q}}\sum_{\mathbf{q}'}\sum_{\mathbf{k}} M^W(\mathbf{k},\omega_{\mathbf{q},\mathbf{k}})C^\dagger_{\mathbf{q}'}C_{\mathbf{q}'-\mathbf{k}}a_{\mathbf{k}}a^\dagger_{\mathbf{k}}C^\dagger_{\mathbf{q}-\mathbf{k}}C_{\mathbf{q}}. \tag{26}$$

As is evident from Eq. (23), there is a possibility that the matrix element becomes negative if energies of dust particulates are close enough, or

$$\omega_{\mathbf{q},\mathbf{k}}^2<\omega_{\mathbf{k}}^2. \tag{27}$$

Although the possible negative matrix element is in $\mathbf{k}$ space, it opens the possibility of negative interaction in real space or attraction between dust particulates [24]. It was indeed shown that the Hamiltonian derived earlier based on a canonical transformation describes the presence of attractive interaction in a form of the wake potential [7]. The potential responsible for the attraction of dust particulates was derived earlier as [6]

$$\phi_W(\mathbf{x},t)=\frac{4\pi Q}{V}\sum_{\mathbf{k}}\frac{\lambda_D^2}{1+k^2\lambda_D^2}\frac{\omega_{\mathbf{k}}^2}{k_z^2v_d^2-\omega_{\mathbf{k}}^2}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}_0-\mathbf{v}t)}, \tag{28}$$

where a dust particulate is located at $\mathbf{x}=\mathbf{x}_0$ at $t=0$ and moves with velocity $\mathbf{v}=v\mathbf{e}_z$ in the ion flow of velocity $\mathbf{v}_0=v_0\mathbf{e}_z$, and $v_d=|v-v_0|$. When $\omega_{\mathbf{q},\mathbf{k}}=k_zv_d$ in Eq. (23), the matrix element is in agreement with the expression in Eq. (28), or

$$Q\phi_W(\mathbf{x},t)=\sum_{\mathbf{k}} M^W(\mathbf{k},k_zv_d)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}_0-\mathbf{v}t)}. \tag{29}$$

IV. HYDRODYNAMIC EQUATION FOR A GAS OF DUST PARTICULATES

In this section, we consider a collection of dust particulates which interact among themselves through the exchange of phonons. The collection of dust particulates is treated as a gas of dust particulates, and is characterized by the perturbed velocity $\mathbf{V}_d$ and the density

$$N_d=N_{d0}+\delta N_d, \tag{30}$$

where δN_d is the perturbation from the equilibrium density N_{d0}. The linearized hydrodynamic equations to describe the dynamics of the gas of dust particulates are

$$\frac{\partial\delta N_d}{\partial t}=-N_{d0}\nabla\cdot\mathbf{V}_d, \tag{31}$$

$$\frac{\partial\mathbf{V}_d}{\partial t}=-\frac{1}{m_d}\nabla\Phi(\mathbf{x},t), \tag{32}$$

where the potential energy $\Phi(\mathbf{x},t)$ is given by

$$\Phi(\mathbf{x},t)=\int M(\mathbf{x}-\mathbf{x}',t-t')\,\delta N_d(\mathbf{x}',t')\,d^3x'\,dt', \tag{33}$$

and $M(\mathbf{x},t)$ is the inverse Fourier transform of a vertex function $M(\mathbf{k},\omega)$. We assume that

$$\delta N_d,\mathbf{V}_d,\Phi\sim e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}; \tag{34}$$

then Eqs. (31), (32), and (33) become

$$-i\omega\,\delta N_d=-iN_{d0}\mathbf{k}\cdot\mathbf{V}_d, \tag{35}$$

$$-i\omega\mathbf{V}_d=-\frac{1}{m_d}i\mathbf{k}\Phi(\mathbf{k},\omega), \tag{36}$$

$$\Phi(\mathbf{k},\omega)=VM(\mathbf{k},\omega)\,\delta N_d. \tag{37}$$

First, consider the case where a vertex function is given by Eq. (24), or

$$M(\mathbf{k},\omega)=M^W(\mathbf{k},\omega). \tag{38}$$

Equations (35)–(37), with Eq. (38), will provide a dispersion relation

$$\omega^4-\omega_{\mathbf{k}}^2\omega^2-\Omega_{\mathbf{k}}^2\omega_{\mathbf{k}}^2=0, \tag{39}$$

where $\omega_{\mathbf{k}}$ is the ion acoustic frequency

$$\omega_{\mathbf{k}}=\frac{k\lambda_D}{\sqrt{1+k^2\lambda_D^2}}\omega_{\mathrm{pi}}, \tag{40}$$

$\Omega_{\mathbf{k}}$ is the dust acoustic frequency

$$\Omega_{\mathbf{k}}=\frac{k\lambda_D}{\sqrt{1+k^2\lambda_D^2}}\omega_{\mathrm{pd}}, \tag{41}$$

and $\omega_{\rm pd}=\sqrt{4\pi N_{d0}Q^2/m_d}$ is the dust plasma frequency. The dispersion relation has approximate solutions

$$\omega_{1,2}=\pm\omega_{\mathbf{k}}\left(1+\frac{\Omega_{\mathbf{k}}^2}{2\omega_{\mathbf{k}}^2}\right)\approx\pm\omega_{\mathbf{k}}, \tag{42}$$

$$\omega_{3,4}=\pm i\Omega_{\mathbf{k}}\left(1-\frac{\Omega_{\mathbf{k}}^2}{2\omega_{\mathbf{k}}^2}\right)\approx\pm i\Omega_{\mathbf{k}}. \tag{43}$$

Equation (42) corresponds to the propagation of ion acoustic waves, while Eq. (43) indicates the presence of an instability. The growth rate of the instability is given by the dust acoustic frequency $\Omega_{\mathbf{k}}$. For a typical situation, consider a spherical dust particulate with a radius 2 μm, a mass density 1 g/cm^3, a charge $Q=-10^3e$, and a dust particulate density N_{d0} of 10^3 cm^{-3} in a background electron temperature $T_e=1$ eV and an electron plasma density of 10^9 cm^{-3}. The dust plasma frequency becomes $\omega_{\rm pd}/2\pi=1.5$ Hz. We have, for $k=1.0$ cm^{-1}, $|\omega_{3,4}|\approx 0.22$ rad/s. The e-folding time for this hydrodynamic instability is about 5 s.

Next, consider the case where the Coulomb interaction between dust particulates with Debye shielding is not negligible and the plasma has an ion flow, or

$$M(\mathbf{k},\omega)=M^C(\mathbf{k},\omega)+M^W(\mathbf{k},\omega-\mathbf{k}\cdot\mathbf{v}_0), \tag{44}$$

where the matrix element for the Coulomb interaction with Debye shielding is

$$M^C(\mathbf{k},\omega)=\frac{4\pi Q^2}{V}\frac{\lambda_D^2}{1+k^2\lambda_D^2}. \tag{45}$$

Equations (35)–(37), together with Eq. (44), will provide a dispersion relation

$$(\omega-\mathbf{k}\cdot\mathbf{v}_0)^2\omega^2-\omega_{\mathbf{k}}^2\omega^2-(\omega-\mathbf{k}\cdot\mathbf{v}_0)^2\Omega_{\mathbf{k}}^2=0, \tag{46}$$

which can be rewritten as

$$1+\frac{1}{k^2\lambda_D^2}-\frac{\omega_{\rm pi}^2}{(\omega-\mathbf{k}\cdot\mathbf{v}_0)^2}-\frac{\omega_{\rm pd}^2}{\omega^2}=0. \tag{47}$$

Equation (47) is similar to the dispersion relation of Buneman instability studied in detail by Ishihara, Hirose, and Langdon [26] (see also Peratt [27]). The system becomes unstable when the negative energy wave $\omega=\mathbf{k}\cdot\mathbf{v}_0-\omega_{\rm pi}/\sqrt{1+k^{-2}\lambda_D^{-2}}$ couples with the positive energy wave $\omega=\omega_{\rm pd}/\sqrt{1+k^{-2}\lambda_D^{-2}}$ in the $\mathbf{k}$ space with a condition $\mathbf{k}\cdot\mathbf{v}_0\lesssim kC_s/\sqrt{1+k^2\lambda_D^2}$. The fastest growing mode is characterized by

$$\omega=\frac{1+i\sqrt{3}}{2}\left(\frac{\omega_{\rm pi}}{2\omega_{\rm pd}}\right)^{1/3}\Omega_{\mathbf{k}} \tag{48}$$

at the resonance

$$\mathbf{k}\cdot\mathbf{v}_0=\frac{kC_s}{\sqrt{1+k^2\lambda_D^2}}, \tag{49}$$

where the growth rate shows the sharp peak with a narrow half-width, and the real frequency shows the abrupt change in its value. In the above parameters for an argon plasma, the fastest growing mode has the real frequency $\omega_r/2\pi=1.2$ Hz, and the growth rate $\gamma=13.1$ rad/s. The effect of collisions of charged particles with neutrals associated with this instability was studied by Rosenberg [28].

V. CONCLUSION

We have formulated the matrix element of interaction between dust particulates through the exchange of phonons. The matrix element suggests the presence of an attractive force between highly charged dust particulates, in agreement with earlier work on the wake potential which was considered as a potential produced behind a dust particulate floating in the ion flow. We considered the dynamics of a gas of dust particulates which interact among themselves by exchanging phonons. Hydrodynamic equations predict the instabilities: one grows with time inversely proportional to the dust acoustic frequency, and results in bunching of dust particulates even in the absence of Coulomb interactions; the other grows with time 0.87 $(\omega_{\rm pi}/2\omega_{\rm pd})^{1/3}$ times faster than the dust acoustic time in the presence of dust-dust Coulomb interactions when ions flow in the plasma. The instability characterized by Eq. (48) appears only when $\mathbf{k}\cdot\mathbf{v}_0\lesssim kC_s/\sqrt{1+k^2\lambda_D^2}$, or $M\lesssim 1$, and the system becomes stable for $M>1$ and forms a stable wake potential behind a dust particulate. Such a stable wake potential and the hydrodynamic instability may be relevant to the observed stable plasma crystal and the phase transition from solidlike crystal structure to liquidlike structure in the laboratory plasmas.

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APPENDIX: PHONON FIELDS

We describe the potential $\phi(\mathbf{x},t)$ associated with ion acoustic waves (phonons) in Fourier series in a large box of volume V as

$$\phi(\mathbf{x},t)=\sum_{\mathbf{k}} D_{\mathbf{k}}[a_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}+a_{\mathbf{k}}^{\dagger}e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}}t)}], \tag{A1}$$

where $\omega_{\mathbf{k}}$ is a positive frequency of the ion acoustic wave. The factor $D_{\mathbf{k}}$ will be determined by a consideration of the field energy. A reality condition of the potential $\phi(\mathbf{x},t)$ is obtained by rewriting Eq. (A1) as

$$\phi(\mathbf{x},t)=\sum_{\mathbf{k},k_z>0}\{D_{\mathbf{k}}[a_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}})}+a_{\mathbf{k}}^{\dagger}e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}})}]+D_{-\mathbf{k}}[a_{-\mathbf{k}}e^{-i(\mathbf{k}\cdot\mathbf{x}+\omega_{\mathbf{k}})}+a_{-\mathbf{k}}^{\dagger}e^{i(\mathbf{k}\cdot\mathbf{x}+\omega_{\mathbf{k}})}]\}, \tag{A2}$$

where the term of $D_{-\mathbf{k}}$ comes from the contribution for $k_z<0$, and we set $\mathbf{k}\rightarrow-\mathbf{k}$ (thus changing $k_z>0$). Equating $\phi(\mathbf{x},t)$ and $\phi^*(\mathbf{x},t)$, a complex conjugate of $\phi(\mathbf{x},t)$, we obtain

$$D_{\pm\mathbf{k}} = D^{*}_{\pm\mathbf{k}}. \tag{A3}$$

The time-averaged field energy in the electric field of the longitudinal wave is

$$\int d^3x \, \frac{\langle E^2 \rangle}{8\pi} = \int d^3x \, \frac{\langle |\nabla \phi|^2 \rangle}{8\pi} = \frac{V}{4\pi} \sum_{\mathbf{k}} k^2 |D_{\mathbf{k}}|^2 (a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + a_{\mathbf{k}} a^\dagger_{\mathbf{k}}), \tag{A4}$$

where $\langle \, \rangle$ shows the time average, and we used the relations $\int d^3x \; e^{i(\mathbf{k}_1 - \mathbf{k}_2)\cdot \mathbf{x}} = V \delta_{\mathbf{k}_1,\mathbf{k}_2}$ and $\langle e^{\pm i(\omega_{\mathbf{k}_1} + \omega_{\mathbf{k}_2})t} \rangle = 0$. We note that the wave energy in each $\mathbf{k}$ mode carries $(\partial \omega \epsilon / \partial \omega)_{\omega_{\mathbf{k}}}$ times the electric field energy, since the wave energy in a plasma includes the particle oscillation energy together with the associated electric field energy [20,21], where $\epsilon = \epsilon(\mathbf{k}, \omega)$ is the dielectric function to characterize a plasma. We now introduce a viewpoint of quantum mechanics, and interpret $a_{\mathbf{k}}$ as destruction operator and $a^\dagger_{\mathbf{k}}$ as creation operator of phonons with momentum $\hbar \mathbf{k}$ and energy $S_{\mathbf{k}} \hbar \omega_{\mathbf{k}}$, where $S_{\mathbf{k}} = [(\partial/\partial\omega)\omega\epsilon]_{\omega_{\mathbf{k}}} / |(\partial/\partial\omega)\omega\epsilon|_{\omega_{\mathbf{k}}}$. The operators $a_{\mathbf{k}}$ and $a^\dagger_{\mathbf{k}}$ satisfy the Boson commutation relation [22]

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}}] = a_{\mathbf{k}} a^\dagger_{\mathbf{k}} - a^\dagger_{\mathbf{k}} a_{\mathbf{k}} = 1, \tag{A5}$$

and can be applied to state vectors as

$$a_{\mathbf{k}} |N_{\mathbf{k}}\rangle = \sqrt{N_{\mathbf{k}}} |N_{\mathbf{k}} - 1\rangle, \tag{A6}$$

$$a^\dagger_{\mathbf{k}} |N_{\mathbf{k}}\rangle = \sqrt{N_{\mathbf{k}} + 1} |N_{\mathbf{k}} + 1\rangle, \tag{A7}$$

while the number operator $a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$ has the eigenvalue $N_{\mathbf{k}}$. The factor $D_{\mathbf{k}}$ may be chosen to satisfy

$$\int d^3x \, \frac{\langle E^2 \rangle}{8\pi} = \sum_{\mathbf{k}} \frac{\hbar \omega_{\mathbf{k}}}{\left| \frac{\partial}{\partial \omega} \omega \epsilon \right|_{\omega_{\mathbf{k}}}} \left(N_{\mathbf{k}} + \frac{1}{2} \right). \tag{A8}$$

Thus we obtain

$$D_{\mathbf{k}} = \pm \sqrt{\frac{4\pi\hbar\omega_{\mathbf{k}}}{V k^2 \left| \frac{\partial}{\partial\omega} \omega\epsilon \right|_{\omega_{\mathbf{k}}}}} = \pm \sqrt{\frac{2\pi\hbar\omega_{\mathbf{k}} \lambda_D^2}{V(1 + k^2 \lambda_D^2)}}. \tag{A9}$$

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